Science Teaching 557

Chaos and Fractals

Two Credit Hours

Prerequisites ST 550

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**Course Designation:** Mathematics

**Course Description:** The development of the basic geometry of fractals, using both deterministic and random methods, the mathematical ideas behind chaos, and the connections between the ideas of chaos and fractals, and applications.

**Materials, Readings, and Resources:** There is no required text for the course. Some computer use is required.

**Course Content:**
1. Classic fractals
2. Set theory
3. Metrics
4. The Chaos Game
5. Iterated Function Systems
6. Dimension
7. Iterative maps
8. Chaos
9. Symbol space
10. Chaotic dynamics
11. Julia sets
12. The Mandelbrot set

**Course goals:**
Students will understand the theoretic underpinnings of fractals and chaos, as well as some of their applications. The abstract theory is closely tied to numeric examples generated on the computer.

**Schedule:** Two week course, 9-4 M-F.

**Attendance:** Required.

**Grading:** Grades will be based on homework, projects, and participation.

**Assessment:** Student learning is assessed by student performance on homework and projects.
### A. Teachers will understand and use Mathematics in Problem-Solving.

1. Teachers for grades K-8 will be able to:
   - (a) Use problem-solving approaches to investigate and understand mathematical content.
   - (b) Formulate and solve problems from both mathematical and everyday situations.
   - (c) Identify, select and use appropriate problem-solving strategies; as well as develop and apply their own strategies.
   - (d) Verify and interpret solutions to problems.
   - (e) Use mathematical language and symbolism to model problem situations.

2. Teachers for grades 5-9 will be able to meet the Standards set for K-8 teachers.

3. Teachers for grades 7-12 will be able to:
   - (a) Use a problem solving approach to investigate and understand mathematical concepts.
   - (b) Formulate and solve problems from both mathematical and everyday experiences.
   - (c) Develop their own processes and techniques for solving problems.

### B. Teachers will understand and use Mathematics in Communication.

1. Teachers for grades K-8 will be able to:
   - (a) Identify and define mathematical concepts in a variety of situations.
   - (b) Communicate mathematical ideas both verbally and in writing.
   - (c) Use drawings, discussion, reading, and listening to learn and communicate mathematical ideas.
   - (d) Use a variety of electronic media and manipulatives to explore and communicate mathematical concepts and problem solutions.

2. Teachers for grades 5-9 will be able to meet the Standards set for K-8 teachers.

3. Teachers for grades 7-12 will be able to:
   - (a) Develop skills in both written and oral communication of mathematical concepts.
   - (b) Learn to communicate effectively at various levels of formality and with people who have differing levels of mathematical understanding.

### C. Teachers will understand and use Mathematics in Reasoning.

1. Teachers for grades K-8 will be able to:
   - (a) Describe logical conclusions.
   - (b) Use information sources, models and known facts to explain mathematical thinking.
   - (c) Make and evaluate mathematical conjectures and validate their own mathematical thinking.
   - (d) Recognize and construct logical arguments for mathematical statements, concepts,
and principles.

(e) Apply a variety of reasoning processes to include deductive and inductive reasoning.

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<td>2</td>
<td>Teachers for grades 5-9 will be able to meet the Standards set for K-8 teachers.</td>
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<td>3</td>
<td>Teachers for grades 7-12 will be able to:</td>
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<td>(a) Recognize patterns, make and refine conjectures and definitions, and construct both formal and heuristic proofs.</td>
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<td>(b) Judge the validity of mathematical arguments.</td>
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<td>(c) Formulate counterexamples.</td>
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**D. Teachers will understand and use Mathematical Connections.**

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<td>Teachers for grades K-8 will be able to:</td>
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<td>(a) Show an understanding of the interrelationships within mathematics.</td>
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<td>(b) Connect mathematics to other disciplines and everyday situations.</td>
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<td>3</td>
<td>Teachers for grades 7-12 will be able to:</td>
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<td>(a) Develop an understanding of the interrelationships within mathematics and an appreciation of its unity.</td>
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<td>(b) Understand and appreciate the power of mathematical language and symbolism in the development of mathematical concepts.</td>
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<td>(c) Explore the connections between mathematics and other disciplines.</td>
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<td>(d) Apply mathematics learned in one context to other contexts.</td>
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**E. Teachers will understand and use numbers, and their relationships, systems and theory.**

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<td>Teachers for grades K-8 will be able to:</td>
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<td>(a) Construct number meanings through everyday experiences and the use of physical materials.</td>
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<td>(b) Understand prenumeration concepts.</td>
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<td>(c) Describe and compare ancient and modern numeration systems by relating counting, grouping, and place value concepts.</td>
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<td>(d) Develop number sense.</td>
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<td>(e) Identify different sets of numbers in the real number system.</td>
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<td>(f) Understand representations of numbers, including mixed numbers, fractions, decimals, and scientific notation.</td>
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<td>(g) Demonstrate ability to use models to explore and explain relationships among fractions, decimals, percents, ratios, and proportions.</td>
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<td>(h) Use the relations of equality and inequality.</td>
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<td>3</td>
<td>Teachers for grades 7-12 will be able to: Explore and discuss the properties, relations, and extensions of the real and complex numbers.</td>
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**F. Teachers will understand and use computation and estimation.**

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<td>1</td>
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(a) Model, explain, and develop proficiency with the basic number facts and algorithms, including addition, subtraction, multiplication and division.

(b) Recognize alternative algorithms for the four basic operations.

(c) Select and use computation techniques appropriate to specific problems and determine the reasonableness of solutions.

(d) Use estimation strategies.

(e) Recognize when estimates are appropriate.

(2) Teachers for grades 5-9 will be able to meet the Standards set for K-8 teachers.

(3) Teachers for grades 7-12 will be able to:

(a) Understand and apply numerical computational and estimation techniques and extend them to symbolic expressions.

(b) Use estimation to assess the reasonableness of solutions.

G. Teachers will have a foundation in geometric concepts.

(1) Teachers for grades K-8 will be able to:

(a) Describe, model, draw and classify geometric figures.

(b) Investigate, predict, and describe the results of combining, subdividing and changing shapes.

(c) Develop spatial sense and relationships.

(d) Relate geometric and measurement ideas.

(e) Use geometric concepts and relationships to describe and model mathematical ideas and relationships to the world.

(f) Solve simple problems in two- and three-dimensional geometry involving parallelism, perpendicularity, congruence, similarity, translation, reflection, rotation, symmetry, and incidence.

(2) Teachers for grades 5-9 will be able to meet the Standards set for K-8 teachers.

(3) Teachers for grades 7-12 will be able to:

(a) Understand the role of axiomatic systems in geometry.

(b) Develop both synthetic and algebraic geometric concepts using coordinates and vectors.

(c) Use geometry as a source of mathematical models for a variety of applications.

(d) Employ geometric reasoning as a problem solving strategy.

(e) Model features of the real world using different geometries.

H. Teachers will understand and use measurement.

(1) Teachers for grades K-8 will be able to:

(a) Determine what needs to be measured, select an appropriate unit of measurement, and then select an appropriate tool with which to measure.

(b) Use standard and nonstandard units for measurement to an appropriate degree of accuracy.

(c) Use estimation, informal procedures, and formulas to solve problems involving linear measures, area, volume, mass, and temperature by using both traditional and metric systems.
(2) Teachers for grades 5-9 will be able to meet the Standards set for K-8 teachers.

(3) Teachers for grades 7-12 will be able to:

(a) Identify and use the appropriate units, tools of measurement, and degree of accuracy required in particular problems, making calculations of relative error as necessary.

(b) Apply measurement as a tool in other disciplines

I. Teachers will understand and use statistics and probability.

(1) Teachers for grades K-8 will be able to:

| (a) Collect data from real world experiences or surveys, organize and display data using various charts/graphs manually and by using appropriate technology, analyze and interpret the data, and write convincing arguments based on the data. |
| (b) Solve elementary statistical problems relating to measures of central tendency, measures of dispersion, regression equations, and non-linear regression. |
| (c) Critically examine and analyze data for reliability and validity. |
| (d) Demonstrate an understanding of randomness by conducting sampling experiments. |
| (e) Find experimental and theoretical discrete probabilities using sample spaces, tree diagrams, and other representations. |
| (f) Plan and conduct simulations to determine experimental probabilities. |
| (g) Compute the mathematical expectation of simple games and lotteries. |
| (h) Solve simple problems involving probability, inference, and the testing of hypotheses. |
| (i) Use simple combinations and permutations to solve counting problems. |

(2) Teachers for grades 5-9 will be able to meet the Standards set for K-8 teachers.

(3) Teachers for grades 7-12 will be able to:

(a) Understand measures of central tendency, variability, and correlation.

(b) Collect, display, analyze, and interpret sample data in a variety of situations.

(c) Investigate the role of estimation and probability in statistical analysis.

(d) Use experimental and theoretical probabilities to formulate and solve problems.

(e) Develop strategies for reasoning and making decisions based on uncertainty.

(f) Explore the probabilistic nature of statistical analyses including hypothesis testing, correlation, analysis of variance, and nonparametric methods.

J. Teachers will understand and use patterns and functions.

(1) Teachers for grades K-8 will be able to:

| (a) Recognize, describe, extend, and create a wide variety of patterns. |
| (b) Represent relationships with manipulatives, tables, graphs, verbal and written statements, and formulas. |
| (c) Describe what a function means both intuitively and using formal mathematical language. |
| (d) Demonstrate a basic understanding of classes of functions and their properties; e.g. linear, exponential, polynomial, and periodic. |
### K. Teachers will understand and apply algebraic concepts.

1. Teachers for grades K-8 will be able to:
   (a) Explore and use variables and open sentences to express mathematical relationships.

2. Teachers for grades 5-9 will be able to:
   (b) Solve real world problems involving linear and quadratic equations and inequalities by using traditional techniques and graphing methods that use technology.

3. Teachers for grades 7-12 will be able to:
   (a) Understand and apply the major concepts of linear and abstract algebra.
   (b) Use theoretical results to understand tangible situations.

### L. Teachers will understand and apply concepts of calculus.


2. Teachers for grade 5-9 will be able to:
   (a) Recognize particular types of change such as linear, quadratic and exponential.
   (b) Use graphs, diagrams, charts, physical models, and graphing technology to explore the notions of limit, differentiation, and integration, and interpret the relationships among them.

3. Teachers for grades 7-12 will be able to:
   (a) Investigate the phenomenon of change as a limiting process.
   (b) Explore intuitively and in depth the concepts of limit, continuity, differentiation, and integration.
   (c) Demonstrate an understanding of the underlying theory of analysis.
   (d) Use properties and techniques of calculus to model phenomena in diverse settings.

### M. Teachers will understand and apply discrete processes.


3. Teachers for grades 7-12 will be able to: Understand concepts and applications of discrete mathematics, such as graph theory, recurrence relations, linear programming, difference equations, and combinatorics.

### N. Teachers will understand the use of technology.

1. Teachers for grades K-8 will be able to:
(a) Use calculators and computers to represent mathematical ideas and construct
different representations of mathematical concepts.  

(b) Use calculators and computers to develop and use alternate strategies for solving
problems.  

(c) Use networking and information technologies to solve problems and broaden the
scope of inquiry.  

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(2) Teachers for grades 5-9 will be able to meet the Standards set for K-8 teachers.  

(3) Teachers for grades 7-12 will be able to:  

| (a) Engender a broad array of mathematical modes of thinking through the use of
   powerful computing tools (including function graphers, curve fitters, symbolic
   manipulators, dynamic geometric software, and programming languages).  
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| (b) Use calculators, spreadsheets, and statistical packages to solve problems.  
   | ST 557  

| (c) Use technology to explore probabilities through simulations.  
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| (d) Use graphing calculators and computer algebra systems in the study and
   application of the calculus.  
   | ST 557  

O. Teachers will develop perspectives on mathematics as a human endeavor.  

(1) Teachers for grades K-8 will be able to:  

| (a) Understand the dynamic nature of mathematics and its increasingly significant role
   in social, cultural, and economic development.  
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| (b) Develop an appreciation for the contributions made by various cultures to the
growth and development of mathematical ideas.  
   | ST 557  

| (c) Investigate the contributions made by individuals, both female and male, and from
   a variety of cultures, in the development of ancient, modern, and current mathematical
   topics.  
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| (d) Gain an understanding of the historical development of major school mathematics
   concepts.  
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(2) Teachers for grades 5-9 will be able to meet the Standards set for K-8 teachers.  

(3) Teachers for grades 7-12 will be able to:  

| (a) Explore the dynamic nature of mathematics and its increasingly significant role in
   social, cultural, and economic development.  
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| (b) Gain an understanding of the historical development of major school mathematics
   concepts.  
   | ST 557  

| (c) Understand the historical development of non-Euclidean geometries and the
   questions relating to the parallel postulate involved in this development.  
   | ST 557  

| (d) Develop an appreciation of the contributions made by the various cultures to the
   growth and development of mathematical ideas.  
   | ST 557  

| (e) Investigate the contributions made by individuals, both female and male, and from
   a variety of cultures, in the development of ancient, modern, and current mathematical
   topics.  
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